Week 5 2023

Thursday, February 16, 2023 6:25 PM

Do: $[fg]' = f'g + fg'$ ← write out Product Rule ($f(x)$, $g(x)$)

then: $\int [fg]' dx = \int [f'g + fg'] dx$

$fg = \int gf' \cancel{dx} + \int fg' dx$

$-\int gf' dx \qquad -\int gf' dx$

$fg - \int gf' dx = \int fg' dx$

$uv - \int v\, du = \int u\, dv$ IBP Formula

let $u = f$, $v = g$

$du = f' dx$, $dv = g' dx$

review: $\int \underbrace{x}_{u}\, \underbrace{\sin x\, dx}_{dv}$ ← indefinite integral

$\boxed{\begin{array}{ll} u = x & dv = \sin x\, dx \\ du = dx & v = -\cos x \end{array}}$

$\int u \cdot dv = uv - \int v \cdot du$

$= uv - \int v\, du$

$= x(-\cos x) - \int (-\cos x)\, dx$

$= -x\cos x + \int \cos x\, dx$ easy integral :)

$= -x\cos x + \sin x + C$

review: $\int \underbrace{x}_{u}\, \underbrace{e^{3x}\, dx}_{dv}$

$\boxed{\begin{array}{ll} u = x & dv = e^{3x}\, dx \\ du = dx & v = \frac{1}{3} e^{3x} \end{array}}$

$\int u \cdot dv = uv - \int v \cdot du$

$= uv - \int v\, du$

$= \frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x}\, dx$

$= \frac{1}{3} x e^{3x} - \frac{1}{3} \cdot \frac{1}{3} e^{3x} + C$

$= \boxed{\frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C}$

decide if Substitution Rule or IBP is appropriate:

Do: find $\int x \sin(x^2)\, dx$ ($u = x^2$)

$u = x^2$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$= \int \sin(x^2)\, x\, dx$

$= \int \sin u \cdot \frac{1}{2} du$

$= \frac{1}{2} \int \sin u\, du$

$= -\frac{1}{2} \cos u + C$

$= \boxed{-\frac{1}{2} \cos(x^2) + C}$

$\boxed{(\sin x)' = \cos x}$

$\boxed{(\cos x)' = -\sin x}$

$\boxed{(-\cos x)' = \sin x}$

Do: find $\int \underbrace{x}_{u}\, \underbrace{\sin(2x)\, dx}_{dv}$

$= uv - \int v\, du$

$= -\frac{1}{2} x\cos(2x) + \frac{1}{2} \int \cos(2x)\, dx$

$= -\frac{1}{2} x\cos 2x + \frac{1}{2} \cdot \frac{1}{2} \sin(2x) + C$

$= \boxed{-\frac{1}{2} x\cos(2x) + \frac{1}{4} \sin(2x) + C}$

$u = x$

$du = dx$

$dv = \sin 2x\, dx$

$v = -\frac{1}{2} \cos 2x$

$\int \sin(2x)\, dx$

$= \frac{1}{2} \int \sin u\, du$

$= -\frac{1}{2} \cos u$

$u = 2x$

$\frac{1}{2} du = \frac{2}{2} dx$

$= \boxed{-\frac{1}{2}\cos(x^2) + C}$

$= \boxed{-\frac{1}{2}\cos(x^2) + C}$ $\qquad = \boxed{-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C}$

Using IBP More Than Once:

ex. find $\int t^2 e^t\,dt \xrightarrow{\text{IBP \#1}} \boxed{\begin{array}{ll} u = t^2 & dv = e^t dt \\ du = 2t\,dt & v = e^t \end{array}}$

$\int u \cdot dv = uv - \int v \cdot du$

$= uv - \int v\,du$

$= t^2 e^t - \int e^t \cdot 2t\,dt$

$= t^2 e^t - 2\underbrace{\int t e^t\,dt}_{u_2 v_2 - \int v_2 du_2} \xrightarrow{\text{IBP \#2}}$ $\quad u_2 = t \quad dv_2 = e^t dt$; $\quad du_2 = dt \quad v_2 = e^t$

$= t^2 e^t - 2\left(te^t - \int e^t\,dt\right)$

$= \boxed{t^2 e^t - 2te^t + 2e^t + C}$

$\int \ln x\,dx = ?$

also $\frac{x^6 \ln x}{6}$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

ex: evaluate $\int x^5 \ln x\,dx$ $\qquad u = \ln x \quad dv = x^5 dx$; $\quad du = \frac{1}{x}dx \quad v = \frac{x^6}{6}$

$\int u \cdot dv = uv - \int v \cdot du$

reorder $= \int \underbrace{\ln x}_{u} \cdot \underbrace{x^5 dx}_{dv}$ $\quad = \underbrace{\frac{1}{6}x^6 \ln x}_{uv} - \frac{1}{6}\int x^6 \cdot \frac{1}{x}\,dx$ $\qquad x^6 \cdot \frac{1}{x} = \frac{x^6}{x} = x^5$

$= \frac{1}{6}x^6 \ln x - \frac{1}{6}\int x^5\,dx$

$= \frac{1}{6}x^6 \ln x - \frac{1}{6}\cdot\frac{1}{6}x^6 + C$ $\qquad$ also

$= \boxed{\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C}$ $\qquad \frac{x^6}{36}$

definite integral

ex.

evaluate $\int_0^1 \underbrace{\arctan x}_{u}\,\underbrace{dx}_{dv}$ $\qquad u = \arctan x \quad dv = dx$; $\quad du = \frac{1}{1+x^2}dx \quad v = x$

$\int u \cdot dv = uv - \int v \cdot du$

$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$

$= x\arctan x\Big|_0^1 - \int_0^1 \frac{1}{1+x^2}x\,dx$

$\int_{0=x}^{1=x} \frac{1}{u}\cdot\frac{1}{2}du$ $\qquad u = 1+x^2$; $\quad du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$= x\arctan x\Big|_0^1 - \frac{1}{2}\int_1^2 \frac{1}{u}\,du\Big|_1^2$ $\qquad u_{x=0}: 1+0^2 = 1$; $\quad u_{x=1}: 1+1^2 = 2$

$\ln|u|$

$= x\arctan x\Big|_0^1 - \frac{1}{2}\ln|u|\Big|_1^2$

$\tan\frac{\pi}{4} = 1$; $\quad \arctan 1 = \frac{\pi}{4}$

$= \underbrace{1\arctan 1}_{\frac{\pi}{4}} - 0\arctan 0 - \frac{1}{2}\left(\ln 2 - \cancel{\ln 1}^{\,0}\right)$

$\quad \frac{\pi}{4} - 0 - \frac{1}{2}\ln 2$ $\qquad = \boxed{\frac{\pi}{4} - \frac{\ln 2}{2}}$

recall from Mon: used u-substitution

$\int x \cdot \sin(x^2)\,dx$ $\qquad$ vs

$= \int \sin(x^2)\,x\,dx \qquad u = x^2$

$$= \int \sin(x^2)\, x\,dx \qquad u = x^2, \quad \tfrac{1}{2}du = x\,dx$$

$$= \int \sin u \left(\tfrac{1}{2}du\right) \qquad u = x^2, \quad \tfrac{1}{2}du = \tfrac{2}{2}x\,dx$$

$$= \tfrac{1}{2}\int \sin u\, du$$

$$= -\tfrac{1}{2}\cos u + C$$

$$= \boxed{-\tfrac{1}{2}\cos(x^2) + C}$$

$(\sin u)' = \cos u$

$(\cos u)' = -\sin u$

$(-\cos u)' = \sin u$

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ex. show $\int \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv} = x\ln x - x + C$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$$= uv - \int v\,du$$

$$= x\ln x - \int x \cdot \tfrac{1}{x}\,dx$$

$$= x\ln x - \int 1\,dx$$

$$= \boxed{x\ln x - x + C}$$

u

L - logarithm

I - inverse

A - algebraic

T - trig

E - exponential

dv

Rule of Thumb - not steadfast

ex. evaluate $\boxed{\int e^x \sin x\,dx} = \int \underbrace{\sin x}_{u}\, \underbrace{e^x\,dx}_{dv}$

$u = \sin x \qquad dv = e^x dx$

$du = \cos x\,dx \qquad v = e^x$

$$= uv - \int v\,du$$

$$= e^x \sin x - \int e^x \cos x\,dx$$

$$= e^x \sin x - \int \cos x\, e^x\,dx \qquad u = \cos x \quad dv = e^x dx \quad du = -\sin x\,dx \quad v = e^x$$

$$\int e^x \sin x\,dx = e^x \sin x - \left(\underbrace{e^x \cos x + \int e^x \sin x\,dx}_{uv - \int v\,du}\right)$$

LIKE TERMS! ᵕ̈

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$+\int e^x \sin x\,dx \qquad\qquad +\int e^x \sin x\,dx$$

$$\tfrac{1}{2}\cdot 2\int e^x \sin x\,dx = \tfrac{1}{2}\left(e^x \sin x - e^x \cos x\right)$$

$$\int e^x \sin x\,dx = \tfrac{1}{2}\left(e^x \sin x - e^x \cos x\right) + C$$

LIATE $\longrightarrow$ dv

Do: find $\int_0^1 (x^2+1)e^{-x}\,dx$

$u = x^2 + 1$

$du = 2x\,dx$

$dv = e^{-x}\,dx$

$v = -e^{-x}$

$v = \int e^{-x}\,dx \qquad u = -x$

$= -\int e^u\,du \qquad -du = dx$

$= -e^u = -e^{-x}$

Do: find $\int_0^1 (x^2+1)e^{-x}\,dx$ $\quad du = 2x\,dx \quad v = -e^{-x}$

$= -e^{-x}(x^2+1)\Big|_0^1 + 2\int_0^1 xe^{-x}\,dx \longrightarrow$ IBP #2 $\quad u = x \quad du = dx \quad dv = e^{-x}dx \quad v = -e^{-x}$

$uv - \int v\,du$

$= -e^{-x}(x^2+1)\Big|_0^1 + 2\left(-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}\,dx\right)$

$= -e^{-x}(x^2+1)\Big|_0^1 - 2xe^{-x}\Big|_0^1 - 2e^{-x}\Big|_0^1$

$= -e^{-1}(1^2+1) - (-e^0(0^2+1)) - 2(1e^{-1} - 0) - 2(e^{-1} - e^0)$

$= -2\cdot\frac{1}{e} + 1 - 2\left(\frac{1}{e}\right) - 2\cdot\frac{1}{e} + 2\cdot 1$

$= 3 - \frac{2}{e}\cdot 3 = \boxed{3 - \frac{6}{e}}$

$\boxed{3 - \frac{6}{e}}$

$e^{-1} = \frac{1}{e}$

use IBP!

$\int \underbrace{x}_{u} \underbrace{\sin(2x)\,dx}_{dv}$

$u = x \qquad dv = \sin(2x)\,dx$

$du = dx \qquad v = \int \sin(2x)\,dx$

$u = 2x$

$\frac{1}{2}du = \frac{2}{2}dx$

$v = \frac{1}{2}\int \sin u\,du$

$= -\frac{1}{2}\cos u$

$v = -\frac{1}{2}\cos(2x)$

$= uv - \int v\,du$

$= -\frac{1}{2}x\cos(2x) - \left(-\frac{1}{2}\right)\int \cos(2x)\,dx$

$= -\frac{1}{2}x\cos(2x) + \frac{1}{2}\cdot\frac{1}{2}\int \cos u\,du \qquad u = 2x \quad \frac{1}{2}du = dx$

$= -\frac{x\cos(2x)}{2} + \frac{1}{4}\sin u + C$

$= \boxed{-\frac{x\cos(2x)}{2} + \frac{1}{4}\sin(2x) + C}$